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Assessing Relative Permeabilities of Two Phase Flows of Water and Steam in Geothermal Reservoirs: State of the Art Relations

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ABSTRACT

A basic understanding of two phase flow of water and steam in geothermal reservoirs is essential to predict the performance of high temperature geothermal wells and reservoirs. Current simulation tools for liquid dominated reservoirs base flow calculations on the traditional Darcy equation, where flow is a function of fluid parameters such as density and viscosity, as well as the intrinsic permeability of the surrounding media to transmit fluid. For two phase flow of water and steam, this approach is based on the relative permeability of each phase, which is the effective portion of the intrinsic permeability for the phase.

The traditional flow relation neglects interfacial shear forces and buoyancy effects acting between the two phases, introducing errors unless the two phases are flowing in completely separated channels. Thus, this formulation predicts that relative permeability is linearly dependent on the water saturation, since it should only account for the portion occupied by that phase in the cross sectional area of the flow channel. Experiments, generally with one dimensional flow, have shown this not to be the case, indicating that the relative permeability scales with the water saturation with an exponent greater than one (Eliasson et al. 1980, Verma 1986, Piquemal 1994, Satik 1998, Mahiya 1999).

In this paper a literature review is presented together with a theoretical analysis of one dimensional two phase flow, using the concept of relative permeabilities as variable functions of water saturation. It is shown that relative permeabilities cannot be material constants but must also depend on flow configuration, thus relationship between water saturation and relative permeabilities must be developed in order to generate predictions more in accordance with observed experimental results. This paper states the theoretical groundwork for a large scale experimental study of the relative permeabilities of two phase flow of water and steam. The goal of the experimental work is to develop empirical relationships for two phase flow, using relative permeabilities, that describe the flow more accurately than existing formulations do.

Theoretical Background

The traditional relation for one and two phase flow in porous media is Darcy’s Law (or Darcy equation). The Darcy equation relates superficial flow velocity to pressure gradient through permeability and viscosity. Using this equation is the conventional way to calculate flow velocity and mass flow in porous media and appears to be adequate for one phase flow where intrinsic permeability of the media is known. For two phase or multi phase flow, the intrinsic permeability cannot be used alone to account for the permeability of the fluid. The relative permeability of each phase must be known, that is the effective permeability of each phase.

For one phase flow, the following Darcy equation (1) expresses the mass flow of a fluid through a porous media.

\[
\dot{m} = \frac{k}{\nu} \left( \frac{\Delta p}{\Delta L} - \rho g \sin \alpha \right)
\]

where \( k \) is the intrinsic permeability of the surrounding media, \( \nu \) is the fluids kinematic viscosity, \( A \) is the cross sectional area of the flow channel, \( \Delta p \) is the pressure loss over a flow channel length \( \Delta L \), \( \rho \) is the fluid density and \( g \) is the earth’s gravity. \( \alpha \) denotes the inclination of the flow channel, \( \alpha = 0^\circ \) (sin \( \alpha = 0 \)) for horizontal flow and \( \alpha = 90^\circ \) (sin \( \alpha = 1 \)) for vertical flow. The unit conventionally used for intrinsic permeability is Darcy, where

\[
1 \text{ Darcy} = 9.8692327 \times 10^{-13} \text{ m}^2
\]

For two phase flows of liquid and gas (such as water and steam) the relative permeability of a phase must be included in the relation as a function of water saturation of the flow. For one phase flow the relative permeability is 1 and is therefore not included in the single phase Darcy equation in equation (1). Equations (3) and (4) show the mass flows for each phase in a two-phase reservoir as represented by the relative permeability approach.

\[
\dot{m}_w = \frac{kk_w(S_w)}{v_w} A \left( \frac{\Delta p}{\Delta L} - \rho_w g \sin \alpha \right)
\]
\[
\dot{m}_s = \frac{k_r w (S_w)}{v_s} A \left( -\frac{\Delta p}{\Delta L} - \rho_w g \sin \alpha \right) \tag{4}
\]
where \( w \) denotes the liquid water portion and \( s \) the steam portion. Water saturation is defined as the area fraction of the liquid phase in the total cross sectional area of both phases,

\[
S_w = \frac{A_w}{A_t}
\tag{5}
\]
and steam saturation is defined as:

\[
S_s = \frac{A_s}{A_t} = 1 - S_w
\tag{6}
\]

\( A_w \) and \( A_s \) are the areas occupied by water and steam respectively, and \( A_t \) is the total cross sectional area of the water and steam flow channel.

To measure the relative permeability, the mass fraction of each phase in the flow must be known. The relative permeabilities can then be calculated as:

\[
k_{rw} = \frac{(1 - x) \dot{m}_t v_w}{k \Delta p / \Delta L - \rho_w g \sin \alpha}
\tag{7}
\]
\[
k_{rs} = \frac{x \dot{m}_t v_s}{k \Delta p / \Delta L - \rho_s g \sin \alpha}
\tag{8}
\]

where \( x \) is the steam quality and \( \dot{m}_t \) is the total mass flow.

In case of a vertical flow, the forces acting on the two phases are resistance force and gravity force (see Equations (3) and (4)). These relations imply that there are no interfacial forces acting between the two phases. Therefore, it can be assumed from the theory that the relative permeabilities are linearly depending on the water saturation before the phases must be flowing in separate channels. For vertical flow, according to Equations (3) and (4), the pressure gradient must be greater than the hydrostatic one for the fluid to flow upwards.

### Measurements of Relative Permeability

Numerous experiments have been performed to study relative permeabilities in two phase flow in porous media. A comparison between a number of experiments is listed in literature (see Horne et al. 2000) where fluid and rock types as well as the saturation measurement techniques are compared. Table 1 summarizes the properties of a number of experiments on relative permeabilities. Figures 1-3 show the relative permeabilities for water and steam from the experiments listed in Table 1 as a function of water saturation. The commonly used Corey curve (Corey 1954) is included in the figures, although it is not represented in Table 1.

Most of the results from the measurements listed in Table 1 and shown in Figures 1-3 do not show linear dependence between water saturation and the relative permeabilities contrary to the expected results from the Darcy equation. An exception from this is the experiment by Ambusso, (Ambusso 1996), where the resulting relative permeabilities vary linearly with saturation. A possible explanation for this can be experimental errors resulting from flow rate determination or faults in the experimental core/rock. (Horne et al. 2000).

The measured relative permeabilities have in some cases been represented as a mathematical function fitting the measured data at the best. In case of the Corey curve, the relative permeabilities are represented as follows (Corey 1954):

\[
k_{rw} = S_w^{4} \tag{9}
\]
\[
k_{rs} = (1 - S_w)^2 (1 - S_s^2) \tag{10}
\]

where the effective or normalized saturation is:

\[
S_{w_n} = (S_w - S_{wr})/(S_{sr} - S_{wr}) \tag{11}
\]

\( S_{wr} \) and \( S_{sr} \) are the limits for water saturation before water and steam become mobile.

Another mathematical function was represented by Verma (Verma 1986) where:

\[
k_{rw} = S_w^3 \tag{12}
\]
\[
k_{rs} = 1.2984 - 1.9832 S_{w_n} + 0.7432 S_{w_n}^2 \tag{13}
\]

and:

\[
S_{w_n} = (S_w - S_{wr})/(S_{sr} - S_{wr}) \tag{14}
\]

where \( S_{sr}=0.2 \) and \( S_{sr}=0.895 \) determine the mobile limits for the phases.

The function fitting the results from Mahiya (Horne et al. 2000) is the following:

\[
k_{rw} = 0.49 \frac{(S_w - S_{wr})^{2.655}}{S_{sr} - S_{wr}} \tag{15}
\]
\[
k_{rs} = 0.63 \frac{(S_{sr} - S_w)^{2.04}}{S_{sr} - S_{wr}} \tag{16}
\]

where \( S_{sr}=0.27 \) and \( S_{sr}=0.87 \).

These functions give a good idea of the correlation between the relative permeabilities and the water saturation. Nevertheless, there seems to be no uniform solution for the relative permeability as a function of water saturation available at this point.

### Effect of Flow Configuration

Up to this point it seems like the problem lies in finding appropriate relations for \( k_w \) and \( k_s \), as functions of water saturation. By eliminating the pressure decline \( \Delta p/\Delta L \) in Equations (3) and (4) the following applies:
By inserting the phase fractions of the total flow as in equations (7) and (8) following relation is gained:

$$\frac{\Delta p}{\Delta L} = \frac{m_w v_w}{k_{rw} A} + \rho_w g \sin \alpha = \frac{m_s v_s}{k_{rs} A} + \rho_s g \sin \alpha$$  \hspace{1cm} (17)

By inserting the phase fractions of the total flow as in equations (7) and (8) following relation is gained:

$$\frac{1}{k_{rw}} = \frac{1}{k_{rs} (1-x)} v_w +$$

$$\frac{g \kappa A}{(1-x) m_i v_w} (\rho_s - \rho_w) \sin \alpha$$  \hspace{1cm} (18)

The relative permeabilities are unknown functions of water saturation, $S_w$.

$$k_{rw} = f(S_w)$$  \hspace{1cm} (19)

$$k_{rs} = g(S_w)$$  \hspace{1cm} (20)

Finding these functions includes solving following equation, derived from equation (18):

$$F(S_w) = \frac{1}{f(S_w)} - \frac{1}{g(S_w)} = 0$$  \hspace{1cm} (21)

where:

$$C_1 = \frac{x v_s}{1-x v_w}$$  \hspace{1cm} (22)

and

$$C_2 = \left(1 - \frac{\rho_s}{\rho_w}\right) \frac{g \kappa A \rho_w}{(1-x) m_i v_w} \sin \alpha$$  \hspace{1cm} (23)

$k_{rw}$ and $k_{rs}$ (and thereby $f(S_w)$ and $g(S_w)$) are values between 0 and 1. These functions must be found in order to go further in solving Equation (21).

According to Equations (3) and (4) the relative permeability for each phase is the same regardless of flow configuration, that is, whether the flow channel is vertical, horizontal or inclined. By comparing the relative permeabilities for the horizontal and vertical case, using Equation (18) and the factors $C_1$ and $C_2$ from Equations (22) and (23) following ratio is gained:

$$\frac{k_{rw, \text{ver}}}{k_{rw, \text{hor}}} = \frac{k_{rs, \text{hor}}}{C_1 k_{rs, \text{hor}}}$$  \hspace{1cm} (24)

If the relative permeabilities were independent of flow configuration, the permeability ratios in Equation (24) would both be 1. The last term can not be zero for all possible flow cases. This leads to the conclusion that the relative permeabilities are not material constants but rather depend on the flow configuration. Finding appropriate relations for $k_{rw}$ and $k_{rs}$ is therefore not the only challenge, the flow relations (Equations (3) and (4)) must also be modified.

Proposed Experimental Apparatus

Many of the past measurements of relative permeabilities of water and steam have in common that they have been performed under horizontal flow conditions and do show deviation from the linear dependency on water saturation, contrary to the expected
Furthermore, results from measurements in a vertical setup show that the two phase fluid can flow upwards although pressure gradient is lower than the hydrostatic force which contradicts the behavior suggested by Equations (3) and (4) (Eliasson et al. 1980).

It is of great importance to perform further measurements in this field, especially for a vertical setup. A new project is underway in collaboration between the University of Iceland and Reykjavik University where relative permeabilities will be measured in a large scale experiment. The results will be used to develop new empirical relationships for two phase flow in geothermal reservoirs and will also be used to improve current simulation tools and used in the construction of a new reservoir modeling tool under development in a connected project.

The planned measurement device consists of a steel pipe, 20 cm in diameter and up to 12 m high. The setup of the pipe can be either horizontal or vertical, but this experiment will focus mainly on the vertical setup. The pipe is designed in modular sections to allow a range of lengths to be examined. The pipe can be filled with various rock types, varying in porosity and intrinsic permeability. Water and steam will be inserted at the bottom of the pipe at pressure which will range up to 100 bar to mimic reservoir condition as well as possible.

Figure 4 shows a schematic setup of the measurement device designed for this project, with different items described in Table 2.

This experiment will offer a new perspective on the theory of two phase flow of water and steam in geothermal reservoirs. It is especially interesting to get information about the behavior of two phase flow in a vertical channel, which previous studies have not extensively covered.

Table 2. Item list for process diagram shown in Figure 4.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Water source</td>
</tr>
<tr>
<td>2.</td>
<td>Flow measurement</td>
</tr>
<tr>
<td>3.</td>
<td>Water pump</td>
</tr>
<tr>
<td>4.</td>
<td>Safety valve</td>
</tr>
<tr>
<td>5.</td>
<td>Exhaust pipeline</td>
</tr>
<tr>
<td>6.</td>
<td>Pressure measurement</td>
</tr>
<tr>
<td>7.</td>
<td>Stop valve</td>
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<tr>
<td>8.</td>
<td>Heater</td>
</tr>
<tr>
<td>9.</td>
<td>Throttling valve</td>
</tr>
<tr>
<td>10.</td>
<td>Inlet measurement</td>
</tr>
<tr>
<td>11.</td>
<td>Experimental apparatus</td>
</tr>
<tr>
<td>12.</td>
<td>Measurements (T,P,vibration)</td>
</tr>
<tr>
<td>13.</td>
<td>Filter</td>
</tr>
<tr>
<td>14.</td>
<td>Outlet measurement (T,P,flow)</td>
</tr>
<tr>
<td>15.</td>
<td>Separator</td>
</tr>
<tr>
<td>16.</td>
<td>Stop valve</td>
</tr>
<tr>
<td>17.</td>
<td>Cooler</td>
</tr>
<tr>
<td>18.</td>
<td>Drain</td>
</tr>
<tr>
<td>19.</td>
<td>Stop valve</td>
</tr>
</tbody>
</table>

Figure 4. Process diagram of the measurement device.

Conclusions

Previous measurements of relative permeability have identified the need for further experiments in this field. According to the literature it is clear that the current relations describing two phase flow of water and steam are insufficient. In most cases, the results from measurements do not show the linear dependency between the relative permeabilities and water saturation as would be predicted by the traditional model. In particular vertical flow is of great interest in order to examine the forces on and between the phases. The relative permeabilities for two phase flow in a vertical flow channel must differ from the ones in a horizontal setup as shown in this paper. It is of great importance to perform further measurements of relative permeabilities in order to improve the current flow relations used in simulation tools for two phase flow of water and steam in geothermal reservoirs.

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References


